(Pages : 4)

C – 1439

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2017
(First Degree Programme Under CBCSS)
MATHEMATICS
MM 1661.1 : Graph Theory (Elective)
(2014 Admn.)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Define parallel edges.
2. A vertex of degree one is called
3. What are vertex disjoint subgraphs ?
4. Define inward demi-degree.
5. Draw a connected graph.
6. If G is an Euler graph then all vertices of G are of _____ degree.
7. Is the following graph an Euler digraph ?



8. How many non-isomorphic trees are there on 4 vertices ?
9. In any tree there are atleast _____ pendent vertices.
10. What is maximal tree of a graph ?

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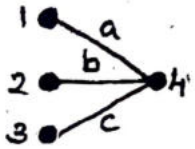
SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Prove that in any graph G, the sum of degrees of all vertices is twice the number of edges.

12. Draw any two graphs having equal number of vertices, edges and same degree sequence, but are non-isomorphic.

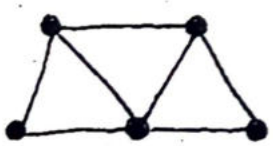
13. Write the incidence matrix of the following graph :



14. Show that the maximum number of edges in a complete bipartite graph on n vertices is $\frac{n^2}{4}$.

15. Prove that, if a graph G has exactly two vertices of odd degree then there must be a path joining these two vertices.

16. Is the following graph Euler ? Explain.



17. Explain Chinese postman problem.

18. Draw all non-isomorphic graphs on 4 vertices.

19. Prove that, if in a graph G there is one and only one path between every pair of vertices, then G is a tree.

20. Prove that a tree with n vertices had n – 1 edges.

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21. Find the center of the following tree :



22. Prove that the distance between vertices of a connected graph is a metric.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry 4 marks **each**.

23. Define complement of a graph, graph isomorphism and self-complementary graph. Give an example of a self-complementary graph.

24. Define spanning subgraph and induced subgraph. Is P_4 a spanning subgraph of K_4 ? Is it an induced subgraph ? Explain.

25. Define adjacency matrix. Draw the graph with the adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

26. In a connected graph G with exactly 2k odd vertices, prove that there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

27. Prove that a simple connected graph has atmost $\frac{(n-1)(n-2)}{2}$ edges.

28. Prove that, given any spanning tree T of a connected graph G with n vertices and e edges has n – 1 tree branches and e – n + 1 chords.

29. Prove that K_5 is non-planar.

30. Draw planar representations of K_4 and $K_{2,3}$.

31. Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

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SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) Show that the maximum number of edges in a simple graph on n vertices is $\frac{n(n-1)}{2}$.

    b) Show that an infinite graph with a finite number of vertices will have atleast one pair of vertices joined by an infinite number of parallel edges.

    c) Sketch 3 non-isomorphic trees on 5 vertices.

33. Explain decanting problem with its graph theoretic formation.

34. a) Prove that there is one and only one path between every pair of vertices in a tree T.

    b) Prove that a graph with n vertices, n – 1 edges and no circuits is connected.

    c) Prove that in any tree T there are atleast two vertices of degree one.

35. Define planar graphs. State a necessary and sufficient condition for a graph G to be planar. Explain Four Colour Theorem and its graph theoretic interpretation.

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